



Radiative mixing of the one Higgs boson and emergent self-interacting dark matter



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ARTICLE INFO

Article history:

Received 12 November 2015

Accepted 18 January 2016

Available online 19 January 2016

Editor: A. Ringwald

ABSTRACT

In all scalar extensions of the standard model of particle interactions, the one Higgs boson responsible for electroweak symmetry breaking always mixes with other neutral scalars at tree level unless a symmetry prevents it. An unexplored important option is that the mixing may be radiative, and thus guaranteed to be small. Two first such examples are discussed. One is based on the soft breaking of the discrete symmetry Z_3 . The other starts with the non-Abelian discrete symmetry A_4 which is then softly broken to Z_3 , and results in the emergence of an interesting dark-matter candidate together with a light mediator for the dark matter to have its own long-range interaction.

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The standard model (SM) of particle interactions requires only one scalar doublet $\Phi = (\phi^+, \phi^0)$ which breaks the electroweak gauge symmetry $SU(2)_L \times U(1)_Y$ spontaneously to electromagnetic $U(1)_Q$ with $\langle \phi^0 \rangle = v = 174$ GeV. It predicts just one physical Higgs boson h , whose properties match well with the 125 GeV particle discovered [1,2] at the Large Hadron Collider (LHC) in 2012. If there are other neutral scalars at the electroweak scale, they are expected to mix with h at tree level, so that the observed 125 GeV particle should not be purely h . If future more precise measurements fail to see deviations, then a theoretical understanding could be that a symmetry exists which distinguishes h from the other scalars. For example, a Z_2 symmetry may exist under which a second scalar doublet (η^+, η^0) is odd [3] and all SM particles are even. This is useful for having a viable dark-matter candidate [4]. It also enables the simplest one-loop (scotogenic) model [5] of radiative neutrino mass through dark matter, if three neutral singlet fermions $N_{1,2,3}$ are also added which are odd under Z_2 , as shown in Fig. 1. Note that this dark Z_2 symmetry may be derived from lepton parity [6].

Another important new development in the understanding of dark matter is the possibility that it has long-range self-interactions [7–12]. This may be an elegant solution to two existing discrepancies in astrophysical observations: (1) central density pro-

files of dwarf galaxies are flatter (core) than predicted (cusp); (2) observed number of satellites in the Milky Way is much smaller than predicted. Although (2) is less of a problem with the recent discovery of faint satellites, there is also (3) the new related problem of predicted massive subhalos which are not observed. From a particle theory perspective, if the light mediator of dark matter is a scalar boson, then it is difficult to construct a theory such that it does not mix substantially with h . In this paper it will be shown how a dark-matter candidate could emerge together with its mediator, such that its mixing with h is one-loop suppressed. This will be discussed later as part of my second example.

If an exact dark symmetry is imposed, then any mixing of h with other neutral scalars in the dark sector would be forbidden. However, if this symmetry is softly broken, it will result in nonzero mixing, many examples of which exist. In the above scotogenic model, if the quadratic term $m_{12}^2 \Phi^\dagger \eta + H.c.$ is added thus breaking Z_2 softly, a nonzero vacuum expectation value $\langle \eta^0 \rangle$ would be induced [13]. This invalidates η^0 as a dark-matter candidate.

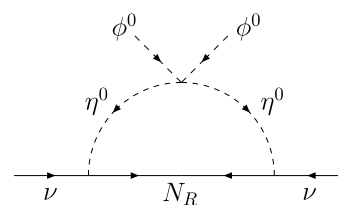


Fig. 1. One-loop Z_2 scotogenic neutrino mass.

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However $\langle \eta^0 \rangle$ could be naturally small as an explanation of the smallness of neutrino masses [13]. At the same time, h mixes with η^0 at tree level as expected. To obtain radiative mixing which has never been discussed before, my first example is the soft breaking of a Z_3 symmetry which would lead to the one-loop mixing of h with two other scalars with zero vacuum expectation values.

Consider the addition of a complex neutral scalar singlet χ to the SM, transforming as $\omega = \exp(2\pi i/3)$ under Z_3 . Let the scalar potential of Φ and χ be given by

$$V = \mu^2 \Phi^\dagger \Phi + m_1^2 \chi^\dagger \chi + \frac{1}{2} m_2^2 \chi^2 + \frac{1}{2} (m_2^*)^2 (\chi^\dagger)^2 + \frac{1}{3} f \chi^3 + \frac{1}{3} f^* (\chi^\dagger)^3 + \frac{1}{2} \lambda_1 (\Phi^\dagger \Phi)^2 + \frac{1}{2} \lambda_2 (\chi^\dagger \chi)^2 + \lambda_3 (\chi^\dagger \chi) (\Phi^\dagger \Phi), \quad (1)$$

where Z_3 is softly broken only by the quadratic m_2^2 and $(m_2^*)^2$ terms. The phase of χ may be redefined to render m_2^2 real, but then f must remain complex in general. Note that Eq. (1) contains all possible Z_3 breaking dimension-two terms, whereas all dimension-three and dimension-four terms respect Z_3 . Consequently, the Z_3 breaking radiative corrections of all dimension-three and dimension-four terms will be finite. On the other hand, if the tree-level dimension-three terms break Z_3 , then the Z_3 breaking dimension-two terms will have infinite radiative corrections, meaning of course that they must be present at tree level in the first place. As it is, Eq. (1) represents a perfectly legitimate and consistent renormalizable field theory.

Consider now the spontaneous breaking of V with $\langle \phi^0 \rangle = v$ and $\langle \chi \rangle = u_R + i u_I$. The minimum of V is given by

$$0 = v[\mu^2 + \lambda_1 v^2 + \lambda_3 (u_R^2 + u_I^2)], \quad (2)$$

$$0 = u_R[(m_1^2 + m_2^2) + \lambda_2 (u_R^2 + u_I^2) + \lambda_3 v^2] + f_R (u_R^2 - u_I^2) - 2 f_I u_R u_I, \quad (3)$$

$$0 = u_I[(m_1^2 - m_2^2) + \lambda_2 (u_R^2 + u_I^2) + \lambda_3 v^2] + f_I (u_I^2 - u_R^2) - 2 f_R u_I u_R. \quad (4)$$

Hence $v \neq 0$ and $u_R = u_I = 0$ are clearly possible solutions within a large space of parameter values. The one Higgs boson h obtains a mass given by $m_h^2 = 2\lambda_1 v^2$, and the 2×2 mass-squared matrix for $\chi = (\chi_R + i\chi_I)/\sqrt{2}$ becomes

$$\mathcal{M}_\chi^2 = \begin{pmatrix} m_1^2 + \lambda_3 v^2 + m_2^2 & 0 \\ 0 & m_1^2 + \lambda_3 v^2 - m_2^2 \end{pmatrix} = \begin{pmatrix} m_R^2 & 0 \\ 0 & m_I^2 \end{pmatrix}. \quad (5)$$

The interaction Lagrangian for h , χ_R , χ_I is then

$$-\mathcal{L}_{int} = \frac{1}{2} m_h^2 h^2 + \frac{1}{\sqrt{2}} \lambda_1 v h^3 + \frac{1}{8} \lambda_1 h^4 + \frac{\lambda_3 v}{\sqrt{2}} h (\chi_R^2 + \chi_I^2) + \frac{\lambda_3}{4} h^2 (\chi_R^2 + \chi_I^2) + \frac{1}{2} m_R^2 \chi_R^2 + \frac{1}{2} m_I^2 \chi_I^2 + \frac{1}{8} \lambda_2 (\chi_R^2 + \chi_I^2)^2 + \frac{f_R}{3\sqrt{2}} \chi_R^3 - \frac{f_I}{\sqrt{2}} \chi_R^2 \chi_I - \frac{f_R}{\sqrt{2}} \chi_R \chi_I^2 + \frac{f_I}{3\sqrt{2}} \chi_I^3. \quad (6)$$

It is clear that h does not mix with χ_R or χ_I at tree level. However there will be radiative mixing in one loop as shown in Fig. 2. The $h\chi_R$ mixing is given by

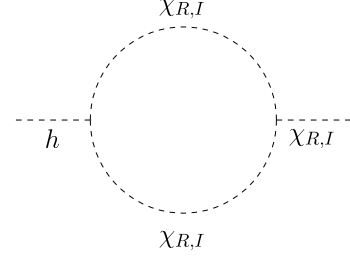


Fig. 2. Radiative mixing of h with $\chi_{R,I}$.

$$2i\lambda_3 f_R v \int \frac{d^4 k}{(2\pi)^4} \left[\frac{1}{(k^2 - m_R^2)^2} - \frac{1}{(k^2 - m_I^2)^2} \right] = \frac{\lambda_3 f_R v}{8\pi^2} \ln \frac{m_R^2}{m_I^2}. \quad (7)$$

Similarly, the $h\chi_I$ mixing is obtained with f_R replaced by f_I and $m_{R,I}$ by $m_{I,R}$. This new phenomenon allows the 125 GeV particle to be essentially h and yet the scalar sector is enriched with other possible consequences different from that of an exact symmetry such as dark parity.

A useful application of the above scenario is in connection with the simplest model [14] of dark matter (DM), i.e. that of a real neutral singlet s , which is odd under dark Z_2 . It has been shown recently [15] that most of its parameter space has been ruled out by the LUX direct-search experiment for dark matter [16]. The tension comes from the requirement that the ss annihilation cross section to be of the correct magnitude to account for the observed DM relic density of the Universe, but its interaction with nuclei through h exchange to be below the LUX bound. To satisfy the latter, the former becomes too small, hence the s relic abundance would exceed what is observed. With the addition of χ , the allowed $ss\chi^\dagger\chi$ interaction would add to the ss annihilation cross section, but would not contribute to the direct-search constraint. In this way the parameter space for s dark matter opens up to $m_s > m_\chi$. This solution also applies to the recently proposed A_4 model [17] of neutrino mass, where $s_{1,2,3} \sim \underline{3}$ and the lightest is dark matter. Here $\chi \sim \underline{1}'$ of A_4 .

My second example of radiative mixing of h and another scalar is based on the breaking of A_4 to Z_3 . Consider now three real neutral scalar singlets $\chi_{1,2,3} \sim \underline{3}$ of A_4 . Let their scalar potential with Φ be given by

$$V = \mu^2 \Phi^\dagger \Phi + \frac{1}{2} m_1^2 (\chi_1^2 + \chi_2^2 + \chi_3^2) + m_2^2 (\chi_1 \chi_2 + \chi_2 \chi_3 + \chi_3 \chi_1) + f \chi_1 \chi_2 \chi_3 + \frac{1}{2} \lambda_1 (\Phi^\dagger \Phi)^2 + \frac{1}{4} \lambda_2 (\chi_1^2 + \chi_2^2 + \chi_3^2)^2 + \frac{1}{4} \lambda_3 (\chi_1^2 \chi_2^2 + \chi_2^2 \chi_3^2 + \chi_3^2 \chi_1^2) + \frac{1}{2} \lambda_4 (\chi_1^2 + \chi_2^2 + \chi_3^2) (\Phi^\dagger \Phi), \quad (8)$$

where A_4 is broken softly to Z_3 by the m_2^2 term. Note that the trilinear f term means that $\chi_{1,2,3}$ do not have the dark parity of the similar $s_{1,2,3}$ scalars discussed previously. Let [18]

$$\begin{pmatrix} \chi_0 \\ \chi \\ \chi^\dagger \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix}, \quad (9)$$

and $\langle \chi_0 \rangle = u_0$, $\langle \chi \rangle = u$, then the minimum of V is given by

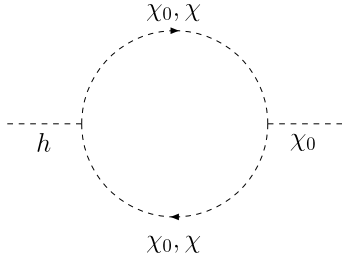


Fig. 3. Radiative mixing of h with χ_0 .

$$0 = v[\mu^2 + \lambda_1 v^2 + \lambda_4(u^2 + u_0^2/2)], \quad (10)$$

$$0 = u_0[m_1^2 + 2m_2^2 + \lambda_4 v^2 + \lambda_2(u_0^2 + 2u^2) + \lambda_3 u_0^2/3 + f u_0/\sqrt{3}] - \lambda_3 u^3/3 - f u^2/\sqrt{3}, \quad (11)$$

$$0 = u[m_1^2 - m_2^2 + \lambda_4 v^2 + \lambda_2(u_0^2 + 2u^2) + \lambda_3(u^2 - u u_0)/2 + f(u - u_0)/\sqrt{3}]. \quad (12)$$

Hence $v \neq 0$ and $u_0 = u = 0$ are again possible solutions within a large space of parameter values. Now the real χ_0 and complex χ are mass eigenstates with $m_0^2 = m_1^2 + \lambda_4 v^2 + 2m_2^2$ and $m_\chi^2 = m_1^2 + \lambda_4 v^2 - m_2^2$. Under the residual Z_3 symmetry, $\chi_0 \sim 1$, $\chi \sim \omega$, and $\chi^\dagger \sim \omega^2$. The interaction Lagrangian for h , χ_0 , χ is then

$$\begin{aligned} -\mathcal{L}_{int} = & \frac{1}{2}m_h^2 h^2 + \frac{1}{\sqrt{2}}\lambda_1 v h^3 + \frac{1}{8}\lambda_1 h^4 \\ & + \frac{\lambda_4 v}{\sqrt{2}}h(\chi_0^2 + 2\chi\chi^\dagger) + \frac{\lambda_4}{4}h^2(\chi_0^2 + 2\chi\chi^\dagger) \\ & + \frac{1}{2}m_0^2\chi_0^2 + m_\chi^2\chi\chi^\dagger + \frac{1}{4}\lambda_2(\chi_0^2 + 2\chi\chi^\dagger)^2 \\ & + \frac{f}{3\sqrt{3}}[\chi_0^3 + \chi^3 + (\chi^\dagger)^3 - 3\chi_0\chi\chi^\dagger] \\ & + \frac{1}{12}\lambda_3[\chi_0^4 - 2\chi_0\chi^3 - 2\chi_0(\chi^\dagger)^3 + 3(\chi\chi^\dagger)^2]. \end{aligned} \quad (13)$$

Again there is no tree-level mixing for h , but radiative mixing occurs between h and χ_0 as shown in Fig. 3. This mixing is given by

$$\begin{aligned} m_{h\chi}^2 = & \sqrt{(2/3)}i\lambda_4 f v \int \frac{d^4 k}{(2\pi)^4} \left[\frac{1}{(k^2 - m_0^2)^2} - \frac{1}{(k^2 - m_\chi^2)^2} \right] \\ = & \frac{\lambda_4 f v}{8\sqrt{6}\pi^2} \ln \frac{m_0^2}{m_\chi^2}. \end{aligned} \quad (14)$$

Hence χ_0 would decay to SM particles through its mixing with h . On the other hand, χ itself is absolutely stable and becomes a suitable DM candidate. It is an example of Z_3 dark matter [19, 20], which may also be connected with the two-loop generation of neutrino mass [21].

The important thing to notice in this model is the natural emergence of χ and χ_0 as the result of A_4 breaking to Z_3 with their accompanying interactions as shown in Eq. (13). Whereas $\chi \sim \omega$ under Z_3 emerges as a dark-matter candidate, $\chi_0 \sim 1$ emerges automatically as its mediator. In other words, all the ingredients of self-interacting dark matter appear at once. Note that this desirable outcome is the result of A_4 breaking to Z_3 . If only Z_3 is assumed in the beginning, then $\chi_0 \sim 1$ would have a substantial tree-level mixing with h . Since the mediator in self-interacting matter is required to be light, say 10 MeV, the fact that its mixing with h is one-loop suppressed by Eq. (14) is an important desirable feature of this model.

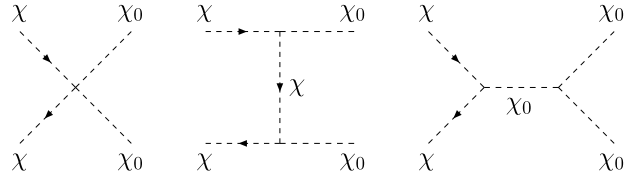


Fig. 4. $\chi\chi^\dagger$ annihilation to χ_0 final states.

Since χ_0 is presumably in thermal equilibrium with all the SM particles, it would be overproduced in the early Universe if it is stable. To avoid any conflict with the very successful predictions of Big Bang Nucleosynthesis (BBN), its decay lifetime should be less than about one second. For $m_{\chi_0} = 10$ MeV, χ_0 would decay to e^-e^+ or to $\gamma\gamma$ through its mixing with h . In contrast to the case of h decay, the former is dominant because the latter lacks the extra enhancement of $m_h^2/m_{\chi_0}^2$. The $\chi_0 \rightarrow e^-e^+$ decay rate is easily calculated to be

$$\Gamma(\chi_0 \rightarrow e^-e^+) = \frac{m_{\chi_0} m_e^2}{16\pi v^2} \left(\frac{m_{h\chi}^2}{m_h^2} \right)^2. \quad (15)$$

For $m_{\chi_0} = 10$ MeV, $m_e = 0.511$ MeV, $v = 174$ GeV, the mixing of χ_0 with h is constrained by $(m_{h\chi}^2/m_h^2) > 2 \times 10^{-5}$ if $\Gamma^{-1} < 1$ s. Using Eq. (14), this translates to

$$\lambda_4 f > 0.02 \text{ GeV}. \quad (16)$$

To be successful as self-interacting dark matter, the trilinear scalar interaction $(f/\sqrt{3})\chi_0\chi\chi^\dagger$ in Eq. (13) should have a magnitude of order the weak scale. Hence the above condition is easily satisfied. On the other hand, the 2×2 mass-squared matrix spanning h and χ_0 is given by

$$\mathcal{M}^2 = \begin{pmatrix} m_h^2 & m_{h\chi}^2 \\ m_{h\chi}^2 & m_0^2 \end{pmatrix}. \quad (17)$$

For $m_0^2 \ll m_h^2$, the $h - \chi_0$ mixing is simply $m_{h\chi}^2/m_h^2$ but the physical mass of χ_0 is also modified, i.e.

$$m_{\chi_0}^2 = m_0^2 - m_{h\chi}^4/m_h^2. \quad (18)$$

For $m_{h\chi}^2/m_h^2 = 2 \times 10^{-5}$, the correction is $(2 \times 10^{-5})^2 (125 \text{ GeV})^2 \simeq 6 \text{ MeV}^2$. If $m_0 = 10$ MeV, m_{χ_0} is reduced only to 9.7 MeV. This is of course acceptable, but if $m_{h\chi}^2/m_h^2$ is too large, m_0 must be delicately adjusted to obtain $m_{\chi_0} \sim 10$ MeV. To avoid too much cancellation, $m_{h\chi}^2/m_h^2 < 10^{-4}$ is a reasonable condition, for which

$$\lambda_4 f < 0.1 \text{ GeV}. \quad (19)$$

Given that $f \sim 100$ GeV is required, $\lambda_4 < 10^{-3}$ is implied. Since $m_0 \simeq 10$ MeV is assumed, $m_1^2 + 2m_2^2 \simeq -\lambda_4 v^2 \sim -30 \text{ GeV}^2$ and $m_1^2 - m_2^2 = m_\chi^2 - \lambda_4 v^2 \sim 10^4 \text{ GeV}^2$, i.e. the fine tuning $m_1^2 \simeq -2m_2^2$ is unavoidable.

The relic abundance of χ is determined by its annihilation cross section as it decouples from other matter in the early Universe. The main interactions are shown in Fig. 4. As a result,

$$\sigma \times v_{rel} = \frac{1}{16\pi m_\chi^2} \left(\lambda_2 - \frac{f^2}{4m_\chi^2} \right)^2. \quad (20)$$

Setting this equal to the optimal value [22] of $4.4 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}$ implied by the observed relic abundance of dark matter in the Universe,

$$\lambda_2 - \frac{f^2}{4m_\chi^2} = 0.0435 \left(\frac{m_\chi}{100 \text{ GeV}} \right) \quad (21)$$

is obtained. The final-state χ_0 particles decay into e^-e^+ pairs through which thermal equilibrium with all other SM particles may be established.

The elastic scattering of χ off nuclei proceeds through h exchange. For $m_\chi = 100 \text{ GeV}$, the LUX bound of 10^{-45} cm^2 implies [15]

$$\lambda_4 < 0.01. \quad (22)$$

This means that direct-search experiments will need to improve by at least one or two orders of magnitude to see this effect. The $h \rightarrow \chi_0\chi_0$ decay rate is given by

$$\Gamma(h \rightarrow \chi_0\chi_0) = \frac{\lambda_4^2 v^2}{16\pi m_h} = \left(\frac{\lambda_4}{0.01} \right)^2 0.5 \text{ MeV}. \quad (23)$$

This would be a contribution to the Higgs invisible width because the lifetime of χ_0 will be certainly long enough for it to escape detection at the LHC.

In conclusion, I have discussed the new idea that scalars beyond the SM may only mix with the one Higgs boson h (identified as the 125 GeV particle discovered at the LHC) in one loop. Two examples have been presented. One is based on the soft breaking of Z_3 . Another is based on the soft breaking of A_4 to Z_3 . In the latter, three real neutral scalars $\chi_{1,2,3}$ are added which transform as $\underline{3}$ under A_4 . With the soft breaking, they are reorganized by Eq. (9) into a real $\chi_0 \sim 1$ and a complex $\chi \sim \omega$ under the residual Z_3 , which is exactly conserved. Whereas χ_0 mixes with h radiatively, χ is a stable dark-matter candidate, with self-interactions mediated by χ_0 . In this way, all the ingredients necessary for self-interacting dark matter emerge together from the softly broken symmetry. The relic abundance of χ is determined by the $\chi\chi^\dagger \rightarrow \chi_0\chi_0$ annihilation

cross section. The direct detection of χ from its elastic scattering off nuclei is possible through h exchange. The light mediator χ_0 decays mainly into e^-e^+ , which may be the source of the observed positron excess in some astrophysical experiments [23,24].

Acknowledgements

This work is supported in part by the U.S. Department of Energy under Grant No. DE-SC0008541.

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